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SUPER ROOT SQUARE MEAN LABELING OF SOME CYCLE RELATED GRAPHS

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ABSTRACT

Let G be a graph with p vertices and q edges. Let $f : V(G) \cup E(G) \rightarrow \{1,2,3,...,p+q\}$ be an injective function. For a vertex labeling f, the induced edge labeling $f^*(e = uv)$ is defined by $f^*(e) = \left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$. Then f is called a super root square mean labeling, if $f(v) \cup f^*(e) = \{1, 2, 3, ...p + q\}$. A graph which admits super root square mean labeling is called super root square mean graph. In this paper, we establish the existence of super root square mean labeling of a few family of cycle related graphs.

Key words: *Super root square mean labeling, regular graph, Union of graphs.*
AMS subject classification(2010): *05C78.*

I. INTRODUCTION

By graph G = (V,E) we mean a finite, undirected graph with neither loops nor multiple edges. The order and size of G are denoted by p and q, respectively. For graph theoretical terminology we refer to Chartrand and Lesniak [3].

By a graph labeling we mean an assignment of numbers to graph elements such as vertices or edges or both subject to some conditions. These conditions are normally expressed on the basis of some values (*weights*) of an evaluating function. Labeling was introduced by SandhyaSet.al[2]. The concept of root square mean labeling was introduced by Thirugnanasambandam and et. al [3].
The following definitions are given for further discussions.

Definition 1.1:

Let $f : V(G) \cup E(G) \rightarrow \{1,2,3,...,p+q\}$ be an injective function. For a vertex labeling f and the induced edge labeling $f^*(e = uv)$ are defined by $f^*(e) = \left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$. Then f is called a super root square mean labeling.

Definition 1.2:
The union of two graphs G_1 and G_2 is a graph S with and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.

Definition 1.3:

The graph $P_n \Theta K_1$ is called a comb.

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Definition 1.4:

The kayak paddle graph, KP(m,n,l) is obtained by joining C_n and C_m by a path of length l.

Definition 1.5:

The graph Flag, Fl_n which is obtained by identifying any one vertex of C_n to an extra vertex is called the root.

Definition 1.6:

The polygonal chain $G_{m,t}$ is a connected graph, all of whose m blocks are polygons C_t .

Definition1.7:

The graph G is obtained by joining a cycle to each of the path and identifying $K_{1,2}$ to the internal vertices of the path.

II. MAIN RESULTS

Theorm2.1

The graph $G = P_m \cup C_n$ admits a super root square mean labeling graph from $m \geq 1$ and $n \geq 3$.

Proof:

Let $V(G) = \{u_i : 1 \leq i \leq m\} \cup \{v_i : 1 \leq i \leq n\}$ and let $E(G) = \{u_i u_{i+1} : 1 \leq i \leq m-1\} \cup \{v_i v_{i+1} : 1 \leq i \leq n-1\}$ be the vertices and edges of G respectively. Then the order and size of G is n+m and n-1+m respectively.

We define $V(G) \cup E(G) \rightarrow \{1, 2, ..., 2(n+m)-1\}$ by

Let $K = \left\lceil \sqrt{\dfrac{1^2 + 4n^2}{2}} \right\rceil$

Case(i):

If K is even

$$\text{Then } f(u_i) = \begin{cases} 2i-1 : 1 \leq i \leq \dfrac{K}{2} \\ 2i : \dfrac{K+1}{2} \leq i \leq n \end{cases}$$

Case(ii):

If K is odd

$$\text{Then } f(u_i) = \begin{cases} 2i-1 : 1 \leq i \leq \dfrac{K-1}{2} \\ 2i : \dfrac{K+1}{2} \leq i \leq n \end{cases}$$

$$f(u_i) = 2n + 2i - 1 : 1 \leq i \leq m$$

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Then the induced edge labels are defined as follows

If K is even

Then $f(u_i u_{i+1}) = \begin{cases} 2i : 1 \le i \le \frac{K-2}{2} \\ 2i+1 : \frac{K}{2} \le i \le n-1 \\ K : i = n \end{cases}$

If K is odd

Then $f(u_i u_{i+1}) = \begin{cases} 2i : 1 \le i \le \frac{K-1}{2} \\ 2i+1 : \frac{K+1}{2} \le i \le n-1 \\ K : i = n \end{cases}$

$f(v_i v_{i+1}) = 2(n+i) : 1 \le i \le m-1$

Thus all the labels of vertices and edges are distinct. Hence the graph G is a super root square mean labeling.

Theorm2.2

The graph $C_n \cup P_m \Theta K_1$ admits a super root square mean labeling for $n \ge 3$ and $m \ge 1$.

Proof:

Let $V(G) = \{u_i : 1 \le i \le n\} \cup \{v_j : 1 \le j \le m\} \cup \{v_j^1 : 1 \le j \le m\}$ and $E(G) = \{e_i : 1 \le i \le m-1\} \cup \{e_j : 1 \le j \le m\} \cup \{e_j^1 : 1 \le j \le m\}$ be the vertices and edges of G respectively. Then the order and size of G is n+2m and n+2m-1 respectively.

We define a map $f : V(G) \cup E(G) \to \{1, 2, ..., 2n+4m-1\}$ by

Let $K = \left\lceil \sqrt{\frac{1^2 + 4n^2}{2}} \right\rceil$

If K is even

Then $f(u_i) = \begin{cases} 2i-1 : 1 \le i \le \frac{K}{2} \\ 2i : \frac{K+1}{2} \le i \le n \end{cases}$

If K is odd

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Then $f(u_i) = \begin{cases} 2i-1 : 1 \le i \le \frac{K-1}{2} \\ 2i : \frac{K+1}{2} \le i \le n \end{cases}$

$f(v_j^1) = \begin{cases} 2n+4j-3 : 1 \le j \le m \\ 2n+4j-1 : 1 \le j \le m \end{cases}$ j is odd

j is even

$f(v_j) = \begin{cases} 2n+4j-1 : 1 \le j \le m \\ 2n+4j-3 : 1 \le j \le m \end{cases}$ j is odd

j is even

Then the induced edge labels are given below

If K is even

Then $f*(e_i) = \begin{cases} 2i : 1 \le i \le \frac{K-2}{2} \\ 2i+1 : \frac{K}{2} \le i \le n-1 \\ K : i = n \end{cases}$

If K is odd

Then $f*(e_i) = \begin{cases} 2i : 1 \le i \le \frac{K-1}{2} \\ 2i+1 : \frac{K+1}{2} \le i \le n-1 \\ K : i = n \end{cases}$

$f*(e_j) = 2n+4j : 1 \le j \le m-1$

$f*(e_j^1) = 2n+4j-2 : 1 \le j \le m$

Thus all the labels of vertices and edges are distinct. Hence the graph G is a super root square mean labeling.

Theorm2.3

Any two regular graphs admit a super root square mean labeling.

Proof:

Let G be a two regular graph. Then the order and size of G are equal.

Let $V(G) = \{u_1, u_2, ..., u_n\}$ and $E(G) = \{e_1, e_2, ..., e_n\}$ be the vertices and edges of G respectively.

Suppose G admits a super root square mean labeling $f:V(G)\cup E(G)\rightarrow\{1,2,...,2n\}$ with $f(u_1)=1$ and $f(u_n)=2n$.

Then $f*(e_n)=\left\lceil\sqrt{\frac{1^2+4n^2}{2}}\right\rceil=K$ (say)

Case(i):

Suppose K is even, then the vertices are labeled with $\{1,3,...,K-1,K+2,...,2n\}$ and the edges are labeled with $\{2,4,...,K-2,K,K+1,...,2n-1\}$.Thus G has all the label of vertices and edges are distinct.

Case(ii):

Suppose K is odd, then the vertices are labeled with $\{1,3,...,K-2,K+1,...,2n\}$ and the edges are labeled with $\{2,4,...,K-3,K,K-1,...,2n-1\}$. Thus G has all the labels of vertices and edges are distinct.

Theorm2.4

Union of 2-regular graph admits a super root square mean labeling.

Proof:

Let $G=\bigcup_{i=1}^{k}G_i$ be a graph where each G_i is a 2-regular graph.
Suppose G admits a super root square mean labeling and
if each G_i of order is distinct, then the order and size of G is $\sum_{i=1}^{k}n_i$

We define $f:V(G)\cup E(G)\rightarrow\left\{1,2,...,2\sum_{i=1}^{k}n_i\right\}$ by

Let $f(u_1^k)=1+2\sum_{i=1}^{k-1}n_i$ and let $f(u_n^k)=2\sum_{i=1}^{k}n_i$

Let $K_k=\left\lceil\frac{f(u_1^k)^2+f(u_n^k)^2}{2}\right\rceil$

If K_k is even, then the corresponding copy of G_k, the sequence of vertices and edges can be labeled as $\left\{1+2\sum_{i=1}^{k-1}n_i,3+2\sum_{i=1}^{k-1}n_i,...,K_k-1,K_k-2,...2\sum_{i=1}^{k}n_i\right\}$ and
$\left\{2+2\sum_{i=1}^{k-1}n_i,4+2\sum_{i=1}^{k-1}n_i,...,K_k-2,K_k+1,...,K_k\right\}$

If K_k is odd , then the corresponding copy of G_k, the sequence of vertices and edges can be labeled as

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$$\left\{1+2\sum_{i=1}^{k-1}n_i, 3+2\sum_{i=1}^{k-1}n_i, ..., K_k-2, K_k+1, ...2\sum_{i=1}^{k}n_i\right\}$$ and

$$\left\{2+2\sum_{i=1}^{k-1}n_i, 4+2\sum_{i=1}^{k-1}n_i, ..., K_k-1, K_k+2, ..., K_k\right\}$$

If each of G_i of order is equal, then the order and size of G is $\sum_{i=1}^{k}n_i$

and the sequence of vertices and edges can be labeled as

If K_k is odd, then

$$\left\{1+2\sum_{i=1}^{k-1}n_i, 3+2\sum_{i=1}^{k-1}n_i, ..., K_k-2, K_k+1, ...2\sum_{i=1}^{k}n_i\right\}$$ and

$$\left\{2+2\sum_{i=1}^{k-1}n_i, 4+2\sum_{i=1}^{k-1}n_i, ..., K_k-1, K_k+2\sum_{i=1}^{k}n_i-1, K_k\right\}$$ respectively.

If K_k is even, then the corresponding copy of G_k , the sequence of vertices and edges can be labeled as

$$\left\{1+\sum_{i=1}^{k-1}n_i, 3+\sum_{i=1}^{k-1}n_i, ..., K_k-1, K_k+2, ...2\sum_{i=1}^{k}n_i\right\}$$ and

$$\left\{2+\sum_{i=1}^{k-1}n_i, 4+\sum_{i=1}^{k-1}n_i, ..., K_k+1, K_k+3, ..., 2\sum_{i=1}^{k}n_i-1, K_k\right\}$$ respectively.

Thus all the labels of vertices and edges are distinct. Hence the theorm.

Theorm2.5

The Graph G(m,n,l) admits super root square mean labeling for m,n $\geq$ 3, l$\geq$ 1.

Proof:

LetV(G)= $\{u_i : 1 \leq i \leq m\} \cup \{v_i : 1 \leq i \leq l\} \cup \{w_i : 1 \leq i \leq n\}$ andE(G)= $\{e_i : 1 \leq i \leq m\} \cup \{e_i^1 : 1 \leq i \leq l+1\} \cup \{e_j : 1 \leq j \leq n\}$ we define f:V(G)$\cup$ E(G)$\rightarrow$ {1,2,3,……….,2(m+n+l)+1)} by

Let K = $\left\lfloor \sqrt{\frac{f(1)^2+f(2n)^2}{2}} \right\rfloor$

Case(i) If K is Odd, then

$$f(u_i) = \begin{cases} 2i-1 : 1 \leq i \leq \frac{K-1}{2} \\ 2i : \frac{K+1}{2} \leq i \leq n \end{cases}$$

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Case(ii) If K is even, then

$$f(u_i) = \begin{cases} 2i-1 : 1 \le i \le \frac{K}{2} \\ 2i : \frac{K+2}{2} \le i \le n \end{cases}$$

$f(v_i)$ = 2m+2i : $1 \le i \le l$

Let $K_1 = \left\lceil \sqrt{\frac{f(w_1^2)^2 + f(u_n^2)^2}{2}} \right\rceil$ let $f(w_i^2) = 2(m+l+2)$

If K_1 and $f(w_1)$ are even then the sequence of vertices of C_n can be labeled as follows,{2(m+l+1), 2(m+l+2), K_1-2, K_1+1,…., 2(m+n+l)+1}

If K_1 is odd and $f(w_1)$ is even, then the sequence of vertices can be labeled as follows, {2(m+l+1), 2(m+l+2), K_1-1, K_1+2,…., 2(m+n+l)+1}

Then the induced edge labels of G are defined as follows

Case(i) If K is Odd, then

$$f*(u_iu_{i+1}) = \begin{cases} 2i : 1 \le i \le \frac{K-1}{2} \\ 2i+1 : \frac{K+1}{2} \le i \le n-2 \\ K : i = n-1 \end{cases}$$

Case(ii) If K is even, then

$$f*(u_iu_{i+1}) = \begin{cases} 2i-1 : 1 \le i \le \frac{K}{2} \\ 2i : \frac{K+2}{2} \le i \le n-1 \\ K = i \le n+3 \end{cases}$$

If K_1 and $f(w_1^2)$ are even then the sequence of edges can be labeled as follows,{2(m+l)+3, 2(m+l)+5, K_1-1, K_1+2,…., K_1 }

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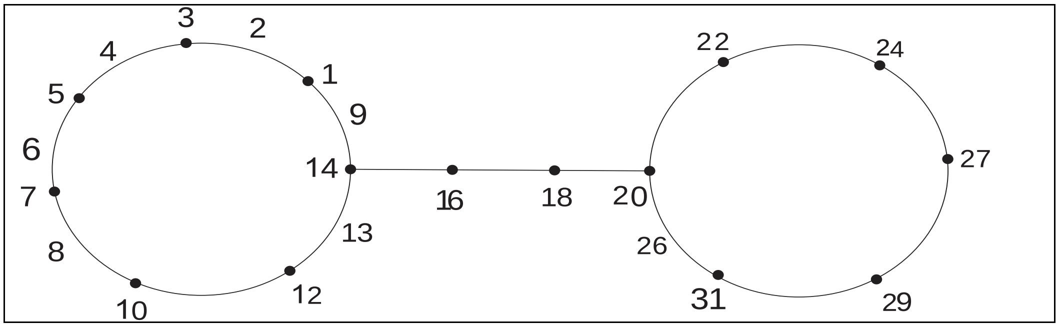
If K_1 is odd and $f(w_1^2)$ is even then the sequence of vertices can be labeled as follows, ,{2(m+l)+3, 2(m+l)+5, K_1-2, K_1+1,…., K_1 }

Thus all the labels of vertices and edges are distinct. Hence the graph G is a super root square mean labeling.

Example 2.6

The super root square mean labeling of KP(7,6,2) is given in figure 1

Figure 1



Theorm2.7

The flag, Fl_n , graph admits a super root square mean labeling for $n \geq 3$.

Proof:

Let V(G) = $\{u_i : 1 \leq i \leq n \cup u\}$ and E(G) = $\{u_i u_{i+1} : 1 \leq i \leq n-1 \cup u\, u_n\}$. Then the order and size of G are n+1 and n+1 respectively.

We define f: $V(G) \cup E(G) \rightarrow \{1, 2, ...2(n+1)\}$ by.

Let K = $\left\lceil \sqrt{\frac{1+4n^2}{2}} \right\rceil$

Let f(u) = 2(n+1)

Case(i) If K is Odd, then

$$f(u_i) = \begin{cases} 2i-1 : 1 \leq i \leq \frac{K-1}{2} \\ 2i : \frac{K+1}{2} \leq i \leq n \end{cases}$$

Case(ii) If K is even, then

$$f(u_i) = \begin{cases} 2i-1 : 1 \le i \le \frac{K}{2} \\ 2i : \frac{K+2}{2} \le i \le n \end{cases}$$

Then the induced edge labels are defined as follows

Case(i) If K is Odd, then

$$f*(u_i u_{i+1}) = \begin{cases} 2i : 1 \le i \le \frac{K-1}{2} \\ 2i+1 : \frac{K+1}{2} \le i \le n-2 \\ K : i = n-1 \end{cases}$$

$$f*(uu_n) = 2n+1$$

Case(ii) If K is even, then

$$f*(u_i u_{i+1}) = \begin{cases} K : i = n-1 \\ 2i : 1 \le i \le \frac{K}{2} \\ 2i+1 : \frac{K+2}{2} \le i \le n-2 \end{cases}$$

Thus all the labels of vertices and edges are distinct. Hence the graph G is a super root square mean labeling.

Example 2.8

The flag Fl_{13} graph on admits a super root square mean labeling is given in figure 2

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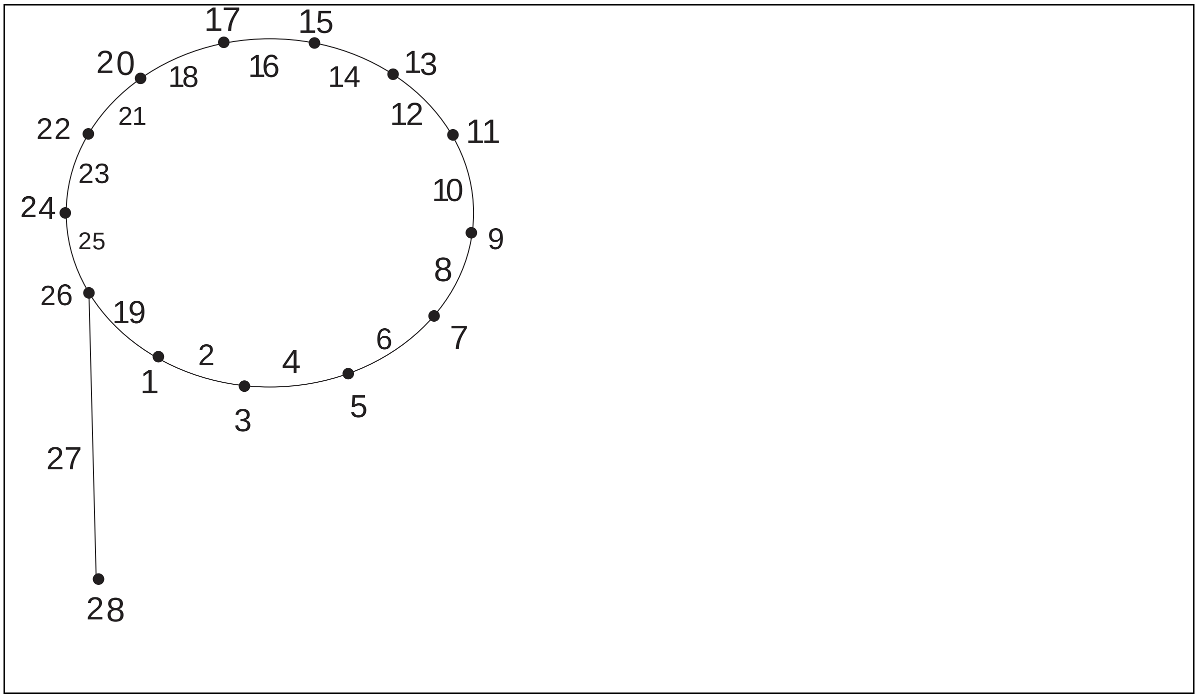
Figure 2



Theorm2.9

The polygonal chain, $C_{n,t}$ admits a super root square mean labeling.

Proof:

Let $V(G)=\{u_i^j : 1\le i\le n, 1\le j\le t\}$ and $E(G)=\{e_i^j, 1\le i\le n, 1\le j\le t\}$ be the vertex and edge set of G respectively. Then the order and size of G is (n-1)t+1 and nt respectively.

We define $f: V(G)\cup E(G) = \{1,2,\ldots\ldots\ldots,t(2n-1)+1\}$.

Let $K_1 = \left\lceil \sqrt{\dfrac{1^2+4n^2}{2}} \right\rceil$

Case(i):

If K_1 is even. Then

$$f(u_i^1) = \begin{cases} 2i-1 : 1\le i\le \dfrac{K_1}{2} \\ 2i : \dfrac{K+2}{2} \le i \le n-1 \end{cases}$$

Case(ii):

If K_1 is odd. Then

$$f(u_i^1) = \begin{cases} 2i-1 : 1 \le i \le \dfrac{K_1 - 1}{2} \\ 2i : \dfrac{K+1}{2} \le i \le n \end{cases}$$

$f(u_i^{j+1} = u_n^j) = 2n + (2n-1)(j-1) : 1 \le j \le t$.

Let $K_i = \left\lceil \sqrt{\dfrac{f(u_i^1)^2 + f(u_{i+1}^1)^2}{2}} \right\rceil$

For $2 \le i \le t$

Case(i) (a):

If K_i is even (odd) and $f(u_i^1)$ is even (odd) and if K_1 is even.. the corresponding cycle of K_i,

$$f(u_j^i) = \begin{cases} 2n + (2n-1)(i-2) + 2(j-1) : 1 \le j \le \dfrac{K_1 - 2}{2}, 1 \le i \le t \\ 2n + (2n-1)(i-2) + 2j - 1 : \dfrac{K_1}{2} \le j \le n. \end{cases}$$

Case(ii)(b):

If K_i is odd(even) and $f(u_i^1)$ is even(odd) and if K (odd). Then the corresponding cycle of K_i

$$f(u_j^i) = \begin{cases} 2n + (2n-1)(i-2) + 2(j-1) : 1 \le j \le \dfrac{K}{2}, 1 \le i \le t \\ 2n + (2n-1)(i-2) + 2j - 1 : \dfrac{K+2}{2} \le j \le n. \end{cases}$$

Thus all the induced edge labels are distinct. Hence the graph $C_{n,t}$ is a super root square mean labeling graph.

Example 2.10

The graph $C_{6,4}$ admits a super root square mean labeling is given in figure 3

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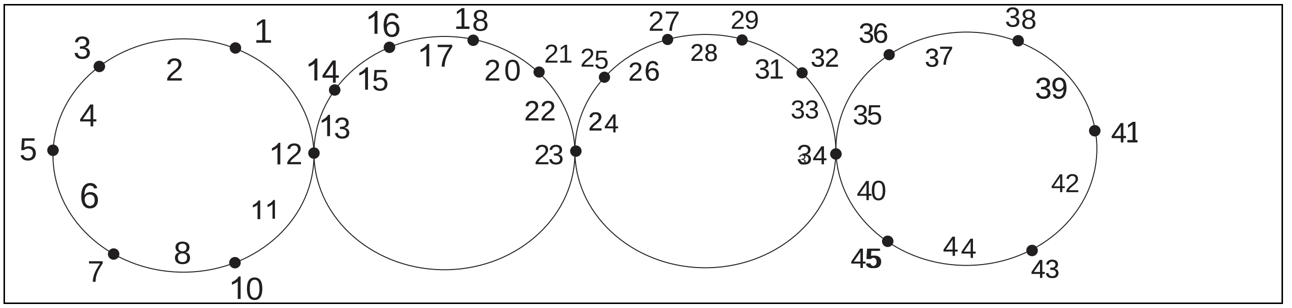
Figure 3



Theorm2.11

KP(m,n,l $K_{1,2}$) is obtained by joining a C_n and C_m by a path of length l and identifying $K_{1,2}$ to the internal vertices of the path.

Proof:

Let $V(G)=\{u_i,1\le i\le n\}\cup\{v_i:1\le i\le m-2\}\cup\{v_i^j:1\le i\le m-2,1\le j\le 2\}\cup\{w_i:1\le i\le n\}$ and $E(G)=\{u_iu_{i+1},1\le i\le n-1\}\cup\{u_n v_1\}\cup\{v_i v_{i+1}:1\le i\le m\}\cup\{w_i w_{i+1}:1\le i\le n-1\}$ be the vertex and edge set of G. Then the order and size of G is 2n+3m-6 and 2n+3m-5 respectively.

We define $f:V(G)\cup E(G)\to\{1,2,.........,4n+6m-11\}$

Let $K_1=\left\lceil\sqrt{\dfrac{1^2+4n^2}{2}}\right\rceil$

If K_1 is even. Then $f(u_i)=\begin{cases}2i-1:1\le i\le\dfrac{K_1}{2}\\ 2i\quad:\dfrac{K_1+2}{2}\le i\le n\end{cases}$

If K_1 is odd. Then $f(u_i)=\begin{cases}2i-1:1\le i\le\dfrac{K_1+1}{2}\\ 2i\quad:\dfrac{K_1-1}{2}\le i\le n\end{cases}$

$f(v_i)=2n+6i-2:1\le i\le m-2$

$f(v_1^1)=2n+1$

$f(v_1^i)=2n+6i-4:2\le i\le m-2$

$f(v_2^i)=2n+6i:1\le i\le m-2$

Let $K_2 = \left\lceil \sqrt{\frac{f(w_1)^2 + f(w_n)^2}{2}} \right\rceil$

If K_1 is even, Then the sequence of vertices can be labeled as $\{2(n+3m-5), 2(n+3m-4), ..., K_2 - 2, K_2 + 1, ...2(n+3m-5)-1\}$

If case K_1 is odd then the sequence of vertices can be labeled as $\{2(n+3m-5), 2(n+3m-4), ..., K_2 - 1, K_2 + 2, ...2(n+3m-5)-1\}$

Then the induced edge labels are defined as follows

If K_1 is even then

$$f(u_i u_{i+1}) = \begin{cases} 2i-1 : 1 \le i \le \frac{K_1}{2} \\ 2i \quad : \frac{K_1+2}{2} \le i \le n-2 \\ K_1 : i = n-1 \end{cases}$$

If K_1 is odd then

$$f(u_i u_{i+1}) = \begin{cases} 2i-1 : 1 \le i \le \frac{K_1 - 1}{2} \\ 2i \quad : \frac{K_1+2}{2} \le i \le n-2 \\ K_1 : i = n-1 \end{cases}$$

$$f(u_n v_1) = 2n+2$$
$$f(v_i v_{i+1}) = 2n+6i-5 : 2 \le i \le m-2$$
$$f(v_i v_1^i) = 2n+6i-3 : 1 \le i \le m-2$$
$$f(v_i v_2^i) = 2n+6i-1 : 1 \le i \le m-2$$

If K_2 is even, then the sequence of edges can be labeled as $\{2(n+3m)-9, (n+3m)-7, ...K_2 - 1, K_2 + 2, ..., K_2\}$

If K_2 is odd, then the sequence of edges can be labeled as $\{2(n+3m)-9, 2(n+3m)-7, ...K_2 - 2, K_2 + 1, ..., K_2\}$

Then all the vertex and edge labels are distinct. Hence the graph G is a super root square mean labeling.

III. CONCLUSION

We have derived that the following graphs are admitted super root square mean labeling $P_m \cup C_n$, $C_n \cup P_m \Theta K_1$, G(m,n,l), G(m,n, K_2)also any two regular graph admits a super root square mean labeling

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